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A Fair Voting Mechanism for Food Distribution Network Design Based on Economic and Sustainability Factors

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Abstract: Governments from developing nations need to build green cities, and food distribution management plays a pivotal role. Distribution network design decisions for setting up warehouses and building road networks need to be based on various economic and sustainability factors. We present a fair voting mechanism to easily choose the best possible set of options by seamlessly considering the different factors. In the process, we make some good contributions to the social choice literature.

Keywords: Sustainability; Distribution network; Social choice; Strategy-proof

1. Introduction to the Research Problem

Consider the situation of a developing nation where the government has money and can accumulate resources to start long-term development projects to achieve its dream of becoming a developed nation. Suppose the government decides to build a green city. The food distribution management system will be an important aspect of city life, and the distribution management system should be socially and environmentally sustainable. Therefore, the locations for constructing distribution centers, warehouses, and transportation links for supplying food products to customers have to be chosen strategically. Still, at the same time, the government will aim to carry out the project in a cost-effective and fair way for all parties involved.

The remainder of the paper is organized as follows. In Section 2, we discuss the background on food distribution management. In Section 3, we describe the research problem in more detail. In Section 4, we discuss the background on social choice theory. In Section 5, we present a fair voting mechanism for our problem setup. In Section 6, we summarize our contributions and provide future research directions.

2. Background on Food Distribution Management

The distribution of food differs significantly from that of other products. Unlike non-perishable goods, food items undergo continuous quality changes throughout the supply chain, right up to the point of consumption. Factors such as limited shelf life, strict temperature and humidity requirements, potential interactions between products, tight delivery time windows, high customer expectations, and narrow profit margins all contribute to the complexity of managing food distribution.

Food supply chains typically extend from agricultural producers to end consumers, often incorporating manufacturing, food service, and retail stages (**Figure 1**). In this context, distribution management generally refers to the movement and storage of products from the final production stage to the end user. As such, it does not encompass the initial segment of the supply chain, specifically, the transport of raw agricultural goods to manufacturers.

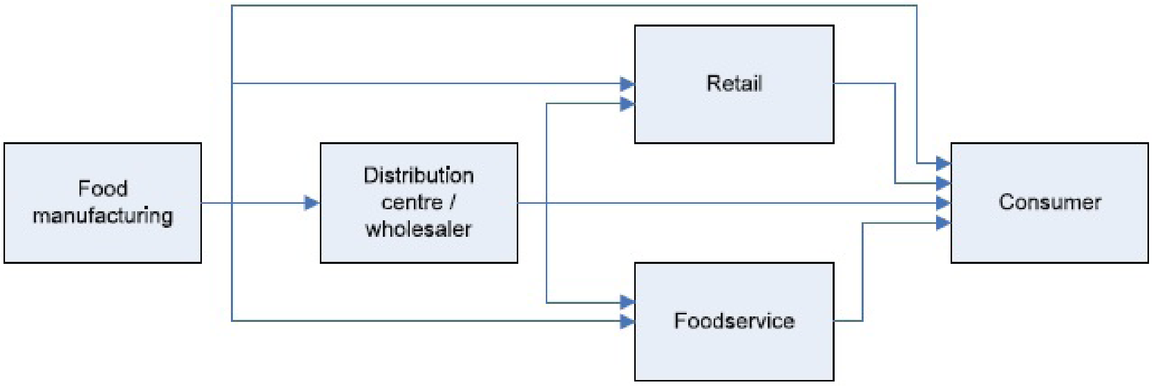

Figure 1. Flow diagram of food supply chain's distribution components. (Akkerman et al. 2010)

In food distribution, three key aspects demand focused attention: food safety, food quality, and sustainability (**Figure 2**).

• Food safety involves preventing illnesses caused by consuming contaminated food, ensuring that all products meet health and hygiene standards throughout the supply chain.

• Food quality encompasses both the physical characteristics of food products and how they are perceived by consumers. This includes factors such as microbial safety, texture, flavor, and overall sensory appeal.

• Sustainability refers to meeting the needs of the current generation without compromising the ability of future generations to meet their own needs, emphasizing responsible resource use and long-term environmental, economic, and social balance. (WCED 1987)

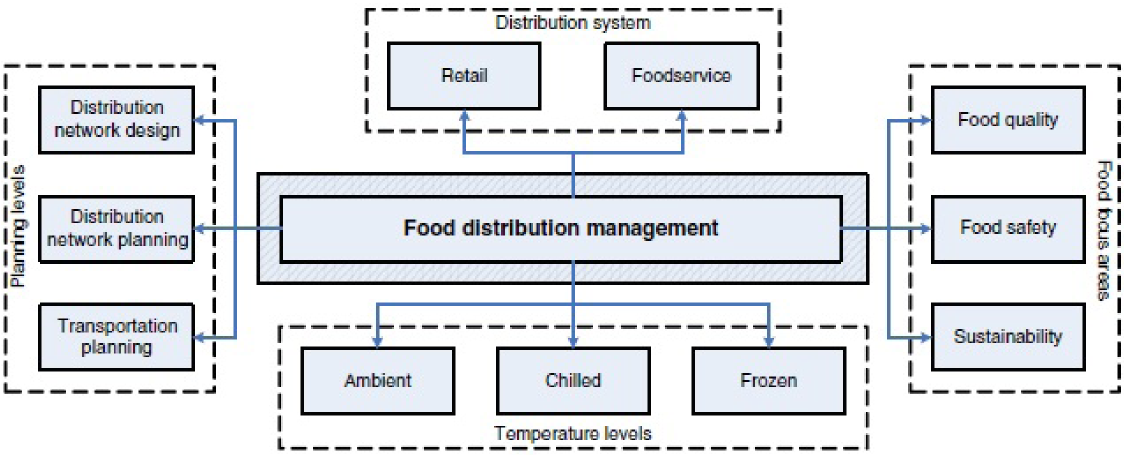

Figure 2. Framework of food distribution management. (Akkerman et al. 2010)

In the literature, food safety and sustainability have not received much attention as compared to food quality. For our research problem the focus will be on sustainability.

Sustainability encompasses both environmental and social dimensions (Kleindorfer et al.

2005). The social aspect includes issues such as employee health and safety, ethical sourcing of raw materials, and animal welfare. For instance, fair trade initiatives aim to enhance the livelihoods of food producers in developing countries. Despite its importance, the social dimension of sustainability has received comparatively less attention in academic literature. From an environmental standpoint, refrigeration used in food storage significantly contributes to environmental impact (Turenne 2009). Additionally, food waste is a critical performance indicator. When food products are discarded instead of consumed, they represent wasted resources and environmental harm without delivering any value. This is often due to spoilage and deterioration over time.

Temperature control is a crucial feature of many food distribution systems. For a wide range of products, maintaining specific temperatures is essential to preserve both food quality and safety. However, this also results in increased energy consumption. As such, temperature-controlled distribution directly impacts all three key aspects of food logistics: quality, safety, and sustainability. In the context of temperature management, food supply chains can generally be categorized into three types: frozen, chilled, and ambient.

In distribution management, decision-making is typically structured across different levels, primarily based on the time horizon of the decisions involved (Anthony 1965, Bitran and Tirupati 1993). This leads to a common distinction between long-term, mid-term, and short-term planning often referred to as strategic, tactical, and operational planning, respectively. Within this hierarchical framework, three distinct planning levels in distribution management are:

• Distribution network design focused on long-term strategic decisions regarding the physical structure of the distribution system. This includes determining the number and size of warehouses and cross-docking facilities, as well as the configuration of transportation links.

• Distribution network planning involves mid-term tactical decisions aimed at meeting forecasted or aggregated demand. Key considerations at this level include planning product flows and determining delivery frequencies.

• Transportation planning deals with short-term operational decisions concerning the actual distribution of customer orders. This includes tasks such as vehicle loading and routing.

For our research problem the focus will be on distribution network design with the element of sustainability in them. Typically, distribution network design are location-allocation problems which are solved using mixed-integer linear programming.

Due to limited attention in the literature, examples of sustainable decision-making processes in food supply chains are relatively scarce. Notably, sustainability is explicitly addressed only in the work of Van der Vorst et al. (2009). Meanwhile, consumer expectations have grown to include not just food quality and safety, but also integrity, sustainability, diversity, and the availability of related information services. As a result, Food Supply Chains (FSCs) must be redesigned to account for all these attributes simultaneously and manage them as effectively as possible. Previous research has often considered these attributes in isolation rather than through an integrated lens. The challenge tackled in the referenced study is to embed models of food quality and sustainability into logistics processes, enabling a comprehensive approach to analyzing logistics, sustainability, and product quality in FSCs. The authors propose a novel simulation environment designed specifically to support the design and redesign of food supply chains. They link travel distances within distribution networks to environmental impacts and demonstrate their approach using a case study on a pineapple supply chain. Two alternative network designs are evaluated based on cost, product quality, energy consumption, and CO_2 emissions. This study is relevant to our research as it represents one of the few that incorporate sustainability into supply chain design. However, our research diverges by focusing more specifically on sustainable distribution network design, with the added objective of integrating fairness as a key consideration.

3. Defining the Research Problem

Research Question: How do we make the distribution network design decisions for food distribution cost-effectively and sustainably while also ensuring fairness (as defined in the Social Choice literature) in the process?

Distribution network design decisions involve setting up distribution centers, warehouses, and roads to deliver food products from manufacturers to retailers.

This involves varying degrees of economic costs. For example, if the warehouses are built in less accessible locations, the costs will be higher. Also, if the roads are built through rough terrain, it will be more costly.

Setting up the warehouses and building the roads will have various other costs. These costs will be from the sustainability perspective, both social and environmental. For example, the Ministry of Natural Resources may impose its costs on specific locations if warehouses or roads are built there based on the amount of natural resources they impact. Also, suppose the warehouses or the roads connecting the warehouses to the retailers are near schools, educational institutions, medical facilities, or residential areas. In that case, they can have a high social cost. Again, there will be an environmental cost based on the distance between the various locations (warehouses, retail locations, *etc*.) of the food distribution management system because the greater the distance, the more the CO_2 emission. This needs to be taken into account because the food distributors will travel these distances on a daily basis. Since the aim is to build a green city, all these different sustainability-based costs are equally important to consider.

To summarize, various criteria can be used to judge and evaluate a location for building a warehouse or an area of land for building a road network. The challenge is to give a single score for each location or area of land so that we can make the distribution network design decisions by holistically considering the costs from all the criteria.

Another equally important issue we need to consider is that the holistic score for a location or area of land can be affected if some ministries or agencies misrepresent their costs to influence the selection process. Therefore, the scoring mechanism should be fair and leave little or no room for misrepresentation.

4. Background on Social Choice Theory

This section is based on the lecture notes of Mishra (2024) which is motivated by the seminal works of Gibbard (1973) and Satterthwaite (1975).

4.1 Strategic Social Choice Theory

4.1.1 Motivation for Strategic Voting

Voting is a fundamental mechanism for collective decision-making and is extensively utilized in various domains of daily life. Common applications include the election of a candidate from a group of nominees, the selection of a project from a set of alternatives within an organization, and the determination of an optimal location for public infrastructure from a finite set of possibilities.

Despite its widespread use, voting processes are often susceptible to strategic manipulation, wherein individuals misrepresent their true preferences to influence the outcome in their favor. Strategic Social Choice Theory seeks to mitigate such behavior by designing mechanisms in which individuals are disincentivized from manipulating the process. Specifically, these mechanisms aim to ensure that truthful revelation of preferences yields the most favorable outcome for each participant, thereby aligning individual incentives with honest behavior.

4.1.2 The Model

- Let $A = \{a, b, c, \ldots, x, y, z, \ldots\}$ be a finite set of alternatives with $|A| = m$.
- Let $N = \{1 \ldots, n\}$ be a finite set of individuals or agents or voters with $|N| = n$.
- Each agent is assumed to possess a preference relation over a finite set of alternatives. Let P_i denote the preference ordering of agent i. We assume that the preference ordering of every agent is a **linear ordering**. Given a preference ordering P_i we say $aP_ib \Leftrightarrow a$ is strictly preferred to b under P_i. Given a preference ordering P_i of agent i, the top ranked element of this ordering (i.e., the most preferred alternative) is denoted by $P_i(1)$, the second ranked element by $P_i(2)$, and so forth, where the ranking continues in decreasing order of preference.

Definition 1 (Ordering). *A preference relation P_i of agent i is referred to as an **ordering** if it satisfies the following properties*:

- *Completeness*: $\forall a,b \in A$ *either* aP_ib *or* bP_ia.
- *Reflexivity*: $\forall a \in A$, aP_ia.
- *Transitivity*: $\forall a,b,c \in A$, $[aP_ib, bP_ic] \Rightarrow [aP_ic]$

Definition 2 (Linear ordering). *A preference relation P_i of agent i is said to be a **linear ordering** if it is an ordering and no two alternatives are indifferent for agent i.*

4.1.3 Social Choice Function

Let $\mathcal{P}$ denote the set of all strict preference orderings over the set of alternatives A. A profile of preference orderings (or simply a preference profile) is denoted as $P \equiv (P_1, \ldots, P_n)$, where each $P_i \in \mathcal{P}$ represents the preference ordering of agent i. Accordingly, the set of all possible preference profiles for n agents is given by $\mathcal{P}^n$.

Definition 3 (SCF). *A* ***social choice function (SCF)*** *is a mapping* $f:\mathcal{P}^n \rightarrow A$.

The following are some desirable properties of an SCF.

Definition 4 (Ontoness). *A SCF f is* ***onto*** *if* $\forall a \in A$ $\exists$ *a preference profile* $P \in \mathcal{P}^n$ *such that* $f(P) = a$.

Definition 5 (Monotonicity). *A SCF f is* ***monotone*** *if for any two profiles P and P′ with* $f(P) = a$ *such that* $\forall b \neq a$*, we have* $aP'_i b$ *if* $aP_i b$ $\forall i \in N$*, then* $f(P') = a$.

It is important to note that, in the definition of monotonicity when we transition from preference profile P to P' with $f(P) = a$, whatever every agent was preferring to a in P continues to prefer it in P' also, but other relations may change. For example, the following is a valid P and P' in the definition of monotonicity with $f(P) = a$ (see **Table 1**).

Table 1. Two valid profiles for monotonicity.

P_1	P_2	P_3	P'_1	P'_2	P'_3
a	b	c	a	a	a
b	a	a	b	c	c
c	c	b	c	b	b

Definition 6 (Efficiency). *A SCF f is* ***efficient*** *if for every preference profile P and* $\forall b \in A$*, if* $\exists a \neq b$ *such that* $aP_i b$ $\forall i \in N$*, then* $f(P) \neq b$. (*Such a SCF is also called Pareto optimal or Pareto efficient or ex-post efficient.*)

The efficiency criterion requires that if all agents strictly prefer an alternative a to another alternative b, then b must not be selected as the outcome.

Definition 7 (Unanimity). *A SCF f is* ***unanimous*** *if for every preference profile* $P \equiv (P_1, \ldots, P_n)$ *with* $P_1(1) = P_2(1) = \cdots = P_n(1) = a$ *we have* $f(P) = a$.

Unanimity represents a fundamental and desirable property in social choice theory. It requires that if all agents unanimously rank a particular alternative as their most preferred option, then this alternative must be selected as the outcome. It is important to note that this formulation of unanimity is stronger than the version which requires the selection of the top-ranked alternative only when the entire *preference ordering* is identical across all agents. In contrast, the definition adopted here mandates agreement solely on the top-ranked alternative, allowing for divergence in the ordering of the remaining alternatives.

Definition 8 (Strategy-Proofness). *A SCF f is* ***manipulable by agent*** *i* ***at profile*** $P \equiv (P_i, P_{-i})$ ***by profile*** $P' \equiv (P'_i, P_{-i})$ *if* $f(P')P_i f(P)$. *A SCF f is strategy-proof if it is not manipulable by any agent i at any profile P by any other profile P′.* (*We use the standard notation* P_{-i} *to denote the preference profile of agents other than agent i, i.e.,* $P_{-i} \in \mathcal{P}^{n-1}$).

This notion of strategy-proofness corresponds to a dominant strategy requirement, as it ensures that no agent can benefit by misrepresenting their preferences, regardless of the preference profiles reported by the other agents.

The following are some examples of SCFs.

Definition 9 (Constant SCF). *A SCF* f^c *is a* ***constant SCF*** *if* $\exists a \in A$ *such that for every preference profile P, we have* $f^c(P) = a$.

Constant SCF satisfies monotonicity and strategy-proofness. However, it fails to satisfy unanimity, onto, or efficiency.

Definition 10 (Dictatorship). *A SCF* f^d *is a* ***dictatorship*** *if* $\exists$ *an agent i, called the dictator, such that for every preference profile P, we have* $f^d(P) = P_i(1)$.

Dictatorship satisfies unanimity, onto, efficiency, monotonicity, and strategy-proofness.

Definition 11 (Plurality SCF with fixed tie-breaking). *Let* $\succ$ *be a linear ordering over alternatives A. For every preference profile P and* $\forall a \in A$*, define the score of a in P as* $s(a,P) = |\{i \in N : P_i(1) = a\}|$. *Define* $\tau(P) = \{a \in A : s(a,P) \geqslant s(b,P)\ \forall\ b \in A\}$ *for every preference profile P, and note that* $\tau(P)$ *is non-empty.*

A SCF f^p *is called a* ***plurality SCF with tie-breaking according to*** $\succ$ *if for every preference profile P,* $f^p(P) = a$*, where* $a \in \tau(P)$ *and* $a \succ b$, $\forall b \in \tau(P) \setminus \{a\}$.

Plurality SCF with fixed tie-breaking satisfies unanimity, onto, and efficiency. However, it fails to satisfy monotonicity or strategy-proofness. To illustrate this, consider an example with three agents $\{1,2,3\}$ and three alternatives $\{a,b,c\}$. Let $\succ$ be defined as: $a \succ b \succ c$. Consider two preference profiles shown in **Table 2**. We note first that $f(P) = a$ and $f(P') = b$. Since bP_3a, agent 3 can manipulate at P by P'.

Table 2. Plurality SCF is manipulable.

P_1	P_2	P_3	$P'_1 = P_1$	$P'_2 = P_2$	P'_3
a	b	c	a	b	b
b	c	b	b	c	a
c	a	a	c	a	c

Definition 12 (Borda SCF with fixed tie-breaking). *Let* $\succ$ *be a linear ordering over alternatives A. Fix a*

preference profile P. $\forall a \in A$, *the rank of a in P for agent i is given by* $r_i(a,P) = k$, *where* $P_i(k) = a$. *From this, the score of alternative a in preference profile P is computed as* $s(a,P) = \sum_{i \in N} [|A| - r_i(a,P)]$. *Define for every preference profile P,* $\tau(P) = \{a \in A : s(a,P) \geqslant s(b,P)\ \forall b \in A\}$.

A SCF f^b *is called* ***a borda SCF with tie-breaking according to*** $\succ$ *if for every preference profile P,* $f^b(P) = a$ *where* $a \in \tau(P)$ *and* $a \succ b$, $\forall b \in \tau(P) \setminus \{a\}$.

Borda SCF with fixed tie-breaking satisfies unanimity, onto, and efficiency. However, it fails to satisfy monotonicity or strategy-proofness. To illustrate this, consider an example with three agents {1,2,3} and three alternatives $\{a,b,c\}$. Let $\succ$ be defined as: $c \succ b \succ a$. Consider two preference profiles shown in **Table 3**. We note first that $f(P) = b$ and $f(P') = c$. Since cP_1b, agent 1 can manipulate at P by P'.

Table 3. Borda SCF is manipulable.

P_1	P_2	P_3	P'_1	$P'_2 = P_2$	$P'_3 = P_3$
a	b	b	c	b	b
c	c	c	a	c	c
b	a	a	b	a	a

The following are some important characterizations of SCFs.

Theorem 1. *A SCF* $f:\mathcal{P}^n \rightarrow A$ *is strategy-proof if and only if it is monotone.*

Lemma 1. *If an SCF f is monotone and onto then it is efficient.*

Lemma 2. *If an SCF f is efficient then it is unanimous.*

Lemma 3. *If an SCF f is unanimous then it is onto.*

The above results can be summarized in the following proposition.

Proposition 1. *Suppose* $f:\mathcal{P}^n \rightarrow A$ *is a strategy-proof SCF. Then, f is onto if and only if it is efficient if and only if it is unanimous.*

The following theorem is the main result of this subsection.

Theorem 2 (Gibbard-Satterthwaite Theorem). *Suppose* $|A| \geqslant 3$. *A SCF* $f:\mathcal{P}^n \rightarrow A$ *is unanimous and strategy-proof if and only if it is a dictatorship.*

4.2 Randomized Strategic Social Choice Theory

4.2.1 Motivation for Randomized Strategic Voting

Lotteries are frequently employed in practical decision-making settings and serve as a natural generalization of deterministic voting outcomes. The incorporation of randomization broadens the set of feasible social choice functions that satisfy strategy-proofness. Consequently, it is of significant theoretical and practical interest to examine the implications of randomization for the property of strategy-proofness.

4.2.2 The Model

- Let $A = \{a,b,c,\ldots,x,y,z,\ldots\}$ be a finite set of alternatives with $|A| = m$.
- Let $N = \{1\ldots,n\}$ be a finite set of individuals or agents or voters with $|N| = n$.
- Let $\Delta(A)$ denote the set of all probability distributions over A. This is the set of **lotteries** over A. A particular element $\lambda \in \Delta(A)$ is a probability distribution over A, and λ_a denotes the probability of alternative a. Of course $\lambda_a \geqslant 0$, $\forall a \in A$ and $\sum_{a \in A} \lambda_a = 1$.

4.2.3 Randomized Social Choice Function

Let $\mathcal{P}^n$ denote the set of all possible profiles of linear orderings for n agents over the set of alternatives A. Let $f_a(P)$ denote the probability of alternative $a \in A$ being chosen at profile $P \in \mathcal{P}^n$.

Definition 13 (RSCF). *A* ***randomized social choice function (RSCF)*** *f is a mapping* $f:\mathcal{P}^n \rightarrow \Delta(A)$.

In this set up, an agent needs to compare two lotteries $\lambda, \lambda' \in \Delta(A)$ given a preference ordering over A.

A utility function $u:A \rightarrow \mathbb{R}$ represents a preference ordering $P_i \in \mathcal{P}$ if $\forall a,b \in A$, $u(a) > u(b) \Leftrightarrow aP_ib$.

Definition 14 (Strategy-Proof RSCF). *An RSCF* $f:\mathcal{P}^n \rightarrow \Delta(A)$ *is* ***strategy-proof*** *if* $\forall i \in N$, $\forall P_{-i} \in \mathcal{P}^{n-1}$, $\forall P_i \in \mathcal{P}$, *and* $\forall$ *utility functions* $u:A \rightarrow \mathbb{R}$ *representing* P_i, *we have* $\sum_{a \in A} u(a) f_a(P_i, P_{-i}) \geqslant \sum_{a \in A} u(a) f_a(P'_i, P_{-i})\ \forall P'_i \in \mathcal{P}$.

An RSCF is said to be strategy-proof if, for every possible preference profile and for every agent, reporting their true preference ordering yields an expected utility that is at least as high as that obtained by misreporting their preferences. It is well established that this notion of strategy-proofness is equivalent to first-order stochastic dominance (FOSD). To define this formally, let $B(a,P_i) = \{b \in A : b = a \text{ or } bP_ia\}$.

Definition 15 (FOSD Strategy-Proof RSCF). *An RSCF* $f:\mathcal{P}^n \rightarrow \Delta(A)$ *is* ***first-order stochastic dominance strategy-proof*** *if* $\forall i \in N$, $\forall P_{-i} \in \mathcal{P}^{n-1}$, $\forall P_i \in \mathcal{P}$, *and* $\forall a \in A$, *we have* $\sum_{b \in B(a,P_i)} f_b(P_i, P_{-i}) \geqslant \sum_{b \in B(a,P_i)} f_b(P'_i, P_{-i})\ \forall P'_i \in \mathcal{P}$.

To understand the definition of FOSD strategy-proof RSCF, let us take an example with two agents {1,2} and three alternatives $\{a,b,c\}$. The preference of agent 2 is fixed at P_2 given by aP_2bP_2c. Let us consider two preference orderings of agent 1: $P_1{:}bP_1cP_1a$ and $P'_1{:}cP_1aP_1b$. Denote $P = (P_1,P_2)$ and $P' = (P'_1,P_2)$. Suppose $f_a(P) = 0.6$ and $f_b(P) = 0.1$ and $f_c(P) = 0.3$. First order stochastic dominance requires the following.

$$f_b(P) = 0.1 \geqslant f_b(P')$$

$$f_b(P)+f_c(P) = 0.4 \geqslant f_b(P')+f_c(P').$$

For f to be manipulable by agent i at profile (P_i,P_{-i}) by profile (P'_i,P_{-i}), $\exists a\in A$ such that

$$\sum_{b\in B(a,P_i)} f_b\left(P_i,P_{-i}\right) < \sum_{b\in B(a,P_i)} f_b\left(P'_i,P_{-i}\right).$$

Definition 16 (Unanimous RSCF). *An RSCF f*: $\mathcal{P}^n \to \Delta(A)$ *satisfies* ***unanimity*** *if* $\forall i\in N$, $\forall P\in\mathcal{P}^n$ *such that* $P_1(1) = P_2(1) = \cdots = P_n(1) = a$, *we have* $f_a(P) = 1$.

Unanimous RSCF is defined in the exact way as the deterministic Unanimous SCF. Therefore, we can see that the Constant SCF is not unanimous even in the randomized case. But there is even a bigger class of RSCFs which are strategy-proof but not unanimous.

Definition 17 (Unilateral RSCF). *An RSCF f is a* ***unilateral*** *if* $\exists$ *an agent i and* $\alpha_1 \geqslant \alpha_2 \geqslant \cdots \geqslant \alpha(|A|)$ *with* $\alpha_j\in[0,1]$ *and* $\sum_{j=1}^{|A|}\alpha_j = 1$ *such that* $\forall P$ *we have* $f_{Pi(j)} = \alpha_j\ \forall j\in\{1,\ldots,|A|\}$.

In a unilateral RSCF, there is a **weak dictator** i such that the outcome probabilities are determined solely by the preference ordering of agent i. The top ranked alternative of i gets probability α_1, second ranked alternative of i gets probability α_2, and so on. Notice that every unilateral RSCF is strategy-proof, but not unanimous.

There is another broad class of RSCFs which are strategy-proof and unanimous.

Definition 18 (Random Dictatorship). *An RSCF f*: $\mathcal{P}^n\to\Delta(A)$ *is a* ***random dictatorship*** *if* $\exists$ *weights* $\beta_1,\ldots,\beta_n\in[0,1]$ *with* $\sum_{i\in N}\beta_i = 1$ *such that* $\forall P\in\mathcal{P}^n$, $f_a(P) = \sum_{i\in N:Pi(1)=a}\beta_i$.

If, in a random dictatorship, a particular agent i is assigned a probability weight $\beta_i = 1$, then the mechanism reduces to a deterministic dictatorship in which agent i is the dictator. More generally, a random dictatorship can be interpreted as a probability distribution over deterministic dictatorships, where β_i denotes the probability with which agent i acts as the dictator. For example, if $N = \{1,2,3\}$ and $A = \{a,b,c\}$ and $\beta_1 = 1/2$, $\beta_2 = \beta_3 = 1/4$, then at a profile P where $P_1(1) = a$, $P_2(1) = a$, $P_3(1) = c$, the output of this random dictatorship will be $f_a(P) = 1/2+1/4 = 3/4$ and $f_c(P) = 1/4$.

Random dictatorship can be viewed as a convex combination of deterministic dictatorships. Given that deterministic dictatorship is both strategy-proof and unanimous, it follows that random dictatorship inherits these properties. The following proposition shows a general result on strategy-proof RSCFs which can be expressed as a convex combination of other strategy-proof RSCFs.

Proposition 2. *Let* $f^1, f^2, \ldots, f^k$ *be a set of k strategy-proof RSCFs. Let* $f{:}\mathcal{P}^n\to\Delta(A)$ *be defined as*: $\forall P\in\mathcal{P}^n$ *and* $\forall a\in A$, $f_a\left(P\right) = \sum_{j=1}^{k}\lambda_j f_a^j\left(P\right)$, *where* $\lambda_j\in[0,1]$ $\forall j\in\{1,\ldots,k\}$ *and* $\sum_{j=1}^{k}\lambda_j = 1$. *Then, f is strategy-proof.*

Another way to interpret the above proposition is that the set of strategy-proof RSCFs form a convex set.

Corollary 1. *Every random dictatorship is strategy-proof.*

The following theorem is the main result of this subsection. This is the counterpart of the Gibbard-Satterthwaite theorem for RSCFs. This was proved by Gibbard.

Theorem 3. *Suppose* $|A| \geqslant 3$. *An RSCF is unanimous and strategy-proof if an only if it is a random dictatorship.*

5. A Fair Voting Mechanism

We present a voting mechanism that is fair and also implementable for our problem setup of making distribution network design decisions for food distribution management based on various economic and sustainability costs. The set of alternatives is various locations for setting up warehouses or regions of land through which the road network can be built to connect the warehouses to the retailers, and the set of agents is the various criteria on which each alternative can be judged. For each criterion, only the preference ordering over the alternatives is sufficient, and there is no need to know the actual cost.

From the previous section, we learned that the desirable properties of unanimity and strategy-proofness give us dictatorship in the deterministic case and random dictatorship in the randomized case. In other words, only the top most preferred alternative matters for each agent. However, if the voting mechanism wants to consider the full ordering of all the alternatives for each agent, we need to consider a weaker version of strategy-proofness.

Definition 19 (*Weakly Manipulable RSCF*). *An RSCF $f:\mathcal{P}^n\to\Delta(A)$ is **weakly manipulable** by agent i at profile $P\equiv(P_i,P_{-i})$ by profile $P'\equiv(P_i',P_{-i})$ if $\forall a\in A$, we have $\sum_{b\in B(a,P_i)} f_b(P') \geqslant \sum_{b\in B(a,P_i)} f_b(P)$ and $\exists a\in A$ such that $\sum_{b\in B(a,P_i)} f_b(P') > \sum_{b\in B(a,P_i)} f_b(P)$.*

Definition 20 (Weakly Strategy-Proof RSCF). *An RSCF $f:\mathcal{P}^n\to\Delta(A)$ is **weakly strategy-proof** if it is not weakly manipulable by any agent i at any profile P by any other profile P′.*

The above definition implies that the set of FOSD strategy-proof RSCFs is a subset of weakly strategy-proof RSCFs. In what follows, we construct a class of RSCFs that satisfy weak strategy-proofness but do not satisfy FOSD strategy-proofness. As a consequence, these functions cannot be characterized as random dictatorship.

Let $|A| = m$ and $s = (s_1,\ldots,s_m)$, where $s_1 \geqslant \cdots \geqslant s_m \geqslant 0$ and $s_1>s_m$. The vector s is called a scoring vector. $\forall i\in N$, $P_i\in\mathcal{P}$, $a\in A$, define the rank of a in P_i as

$$r(a,P_i) = |\{b\in A\backslash\{a\}:bP_ia\}|+1.$$

The score of rank $r(a,P_i)$ is $s_{r(a,P_i)}$. For every profile $P\in\mathcal{P}^n$ compute the score of alternative a as

$$s(a,P) = \sum_{i\in N} s_{r(a,P_i)}.$$

Definition 21 (Normalized Scoring Rule). *An RSCF $f:\mathcal{P}^n\to\Delta(A)$ is a **normalized scoring rule** if for every preference profile P, $\forall a\in A$, we have $f_a(P) = s(a,P)/G$, where $G = \sum_{a\in A} s(a,P) = n\left(\sum_{i=1}^{m} s_i\right)$ and $|N| = n$.*

The following proposition presents a characterization of normalized scoring rules.

Proposition 3. *Normalized scoring rules are FOSD strategy-proof but not unanimous.*

Proof. It is easy to check that normalized scoring rules are not unanimous. If $P_1(1) = P_2(1) = \cdots = P_n(1) = a$, then $s(a,P) = n(s_1) \leqslant G$. Therefore, $f_a(P) \leqslant 1$.

Now to show that normalized scoring rules are FOSD strategy-proof we need to show that $\forall i\in N$, $\forall P_{-i}\in\mathcal{P}^{n-1}$, $\forall P_i\in\mathcal{P}$, and $\forall a\in A$, we have

$$\sum_{b\in B(a,P_i)} f_b(P_i,P_{-i}) \geqslant \sum_{b\in B(a,P_i)} f_b(P_i',P_{-i}) \quad \forall\, P_i'\in\mathcal{P}$$

$$\Leftrightarrow \sum_{b\in B(a,P_i)} s\big(b,(P_i,P_{-i})\big) \geqslant \sum_{b\in B(a,P_i)} s\big(b,(P_i',P_{-i})\big) \quad \forall\, P_i'\in\mathcal{P}$$

$$\Leftrightarrow \sum_{b\in B(a,P_i)} \left(s_{r(b,P_i)} + \sum_{j\in N\backslash\{i\}} s_{r(b,P_j)}\right) \geqslant \sum_{b\in B(a,P_i)} \left(s_{r(b,P_i')} + \sum_{j\in N\backslash\{i\}} s_{r(b,P_j)}\right) \quad \forall\, P_i'\in\mathcal{P}$$

$$\Leftrightarrow \sum_{b\in B(a,P_i)} s_{r(b,P_i)} \geqslant \sum_{b\in B(a,P_i)} s_{r(b,P_i')} \quad \forall\, P_i'\in\mathcal{P}.$$

The above set of equations are satisfied because of the way scoring rules are defined. Therefore, normalized scoring rules are FOSD strategy-proof.

Definition 22 (Unanimous Normalized Scoring Rule). *An RSCF $f:\mathcal{P}^n\to\Delta(A)$ is an **unanimous normalized scoring rule** if f is an unanimous RSCF wherever possible, otherwise f is a normalized scoring rule.*

If the top most preferred alternative of all the agents are same, i.e., $P_1(1) = P_2(1) = \cdots = P_n(1) = a$, then we will apply unanimity, i.e., $f_a(P) = 1$. If P is not of such kind, then f is the normalized scoring rule.

Proposition 4. *Unanimous normalized scoring rules are weakly strategy-proof but not FOSD strategy-proof.*

Proof. We consider the four possible cases depending on the types of the preference profiles $P\equiv(P_i,P_{-i})$ and $P'\equiv(P_i',P_{-i})$.

CASE 1: The top most preferred alternative of all the agents are same in both P and P', i.e., $P_1(1) = \cdots = P_{i-1}(1) = P_i(1) = P_{i+1}(1) = \cdots = P_n(1) = P'_i(1) = c$, say.

Then $f_c(P) = f_c(P') = 1$. Therefore, in this case

$$\sum_{b\in B(a,P_i)} f_b(P_i,P_{-i}) = 1 = \sum_{b\in B(a,P_i)} f_b(P_i',P_{-i}) \quad \forall\, a\in A.$$

Hence, f is FOSD strategy-proof.

CASE 2: The top most preferred alternative of all the agents are not same in both P and P'.

Since we cannot apply unanimity on f in this case, f will act as a normalized scoring rule and, therefore, f is FOSD strategy-proof.

CASE 3: The top most preferred alternative of all the agents are same in P but not in P', i.e., $P_1(1) = \cdots = P_{i-1}(1) = P_i(1) = P_{i+1}(1) = \cdots = P_n(1) = c\neq P'_i(1)$, say.

Then, $f_c(P) = 1>f_c(P')$. Therefore, in this case

$$\sum_{b\in B(a,P_i)} f_b(P_i,P_{-i}) = 1 \geqslant \sum_{b\in B(a,P_i)} f_b(P_i',P_{-i}) \quad \forall\, a\in A.$$

Hence, f is FOSD strategy-proof.

CASE 4: The top most preferred alternative of all the agents are same in P' but not in P, i.e., $P_1(1) = \cdots = P_{i-1}(1) = P'_i(1) = P_{i+1}(1) = \cdots = P_n(1) = c \neq P_i(1)$, say.

Then, $f_c(P) < 1 = f_c(P')$. We have two subcases here depending on the position of c in the preference ordering P_i.

SUBCASE A: $P_i(m) = c$. Therefore, in this case

$$\sum_{b \in B(a,P_i)} f_b\left(P_i, P_{-i}\right) > 0 = \sum_{b \in B(a,P_i)} f_b\left(P'_i, P_{-i}\right) \quad \forall\, a \in A \setminus \{c\}.$$

Hence, f is FOSD strategy-proof.

SUBCASE B: $P_i(k) = c$ where $1 < k < m$. Therefore, in this case

$$\sum_{b \in B(a,P_i)} f_b\left(P_i, P_{-i}\right) > 0 = \sum_{b \in B(a,P_i)} f_b\left(P'_i, P_{-i}\right) \quad \forall\, a \in \left(B\left(c, P_i\right) \setminus \{c\}\right)$$

and

$$\sum_{b \in B(a,P_i)} f_b\left(P_i, P_{-i}\right) \leqslant 1 = \sum_{b \in B(a,P_i)} f_b\left(P'_i, P_{-i}\right) \quad \forall\, a \in A \setminus \left(B\left(c, P_i\right) \setminus \{c\}\right).$$

Hence, f is weakly strategy-proof but not FOSD strategy-proof.

Therefore, the class of unanimous normalized scoring rules are weakly strategy-proof but not FOSD strategy-proof.

6. Conclusions and Future Directions

We presented a fair voting mechanism using social choice theory to help make cost-effective and sustainable distribution network design decisions.

The research problem discussed here is important for food distribution management because we present a framework to account for sustainability (both social and environmental) as easily as other economic factors while making strategic planning decisions. The importance of the work is very well understood when we acknowledge that these factors have not been accounted for in the literature together. Also, sustainability, especially social sustainability, is one of the least explored factors in the food management literature on its own.

Developing novel methodologies to quantify and integrate performance indicators across the various dimensions of sustainability remains a significant challenge for future research. As noted by Kleindorfer et al. (2005), this endeavor requires a comprehensive approach to triple-bottom-line thinking, one that incorporates considerations of profit, people, and the planet into the organizational culture, strategic decision-making, and operational practices of firms.

Regarding our contributions to the social choice literature, we presented a weaker definition of strategy-proofness and also defined a class of RSCFs that satisfy the weaker definition but not the FOSD strategy-proofness definition. This gives us a voting mechanism for which the full ordering of all the alternatives is relevant, not just the top most preferred alternative, for each agent. This voting mechanism can be used for various other applications beyond food distribution network design.

As future research topics in the social choice domain, it will be important to characterize the set of all unanimous and weakly strategy-proof RSCFs and also prove geometric properties like convexity, characterizing extreme points, *etc*.

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