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A Continuous-Time Framework for Time-Weighted Returns, Money-Weighted Returns, Internal Rates of Return, and Dietz Return Estimators

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Abstract: This paper develops a unified continuous-time framework for portfolio return measurement in the presence of external cash flows. Beginning with a stochastic differential equation describing portfolio value dynamics, we demonstrate how several widely used return measures—including the Time-Weighted Return (TWR), Simple Dietz Return, Modified Dietz Return, and Internal Rate of Return (IRR), also known as the Money-Weighted Return (MWR) or Dollar-Weighted Return (DWR)—may be interpreted as alternative reductions or approximations of a common portfolio evolution process.

The framework clarifies the relationships among these methodologies and identifies the role of invested-capital approximation in performance measurement. In particular, the Modified Dietz Return denominator is interpreted as a first-order approximation to average invested capital within a continuous-time setting, providing a theoretical explanation for its practical effectiveness. The analysis further examines the sources of divergence among return measures and demonstrates how differences arise from cash-flow timing, cash-flow magnitude, valuation frequency, and portfolio volatility.

The framework is subsequently extended to AUM fee arrangements, establishing a direct relationship between fee revenue and average invested capital through time. Numerical examples illustrate both the convergence and divergence of the major return measures under alternative cash-flow scenarios.

The principal contribution of this paper is the development of a unified continuous-time interpretation of portfolio return measurement. By identifying average invested capital as a common underlying quantity, the framework clarifies the mathematical and economic relationships among Time-Weighted Returns, Money-Weighted Returns, Internal Rates of Return, Dietz Return estimators, and AUM fee economics.

Keywords: Time-Weighted Return; Money-Weighted Return; Internal Rate of Return; Dollar-Weighted Return; Modified Dietz Return; Portfolio Performance Measurement; Continuous-Time Finance; Assets Under Management (AUM)

1. Introduction

1.1 Motivation

Portfolio performance measurement occupies a central role in investment management, pension administration, wealth management, and fiduciary oversight. Despite its apparent simplicity, the measurement of portfolio returns becomes considerably more complex when external cash flows occur during the evaluation period.

Contributions and withdrawals alter the amount of capital exposed to investment risk and therefore complicate the interpretation of portfolio performance. Over time, several methodologies have emerged to address this problem.

Time-Weighted Returns seek to isolate the investment manager's contribution by eliminating the effects of external cash flows. Money-Weighted Returns incorporate the timing and magnitude of those cash flows and therefore reflect the investor's realized experience. Practical approximations such as the Simple Dietz and Modified Dietz Return methodologies were developed to estimate money-weighted performance when frequent portfolio valuations were unavailable [3],[12].

Although these methodologies are widely used throughout the investment industry, they are frequently presented as distinct and unrelated approaches to performance measurement. As a result, practitioners often understand how to compute these measures without fully understanding their mathematical relationships or the circumstances under which they are expected to agree or diverge.

The existence of multiple return methodologies frequently leads to confusion among investors, consultants, fiduciaries, and investment professionals. Understanding the origins and relationships of these measures therefore remains an important topic in both investment theory and practice [1,4,5].

1.2 Historical Development of Return Measurement

The development of portfolio return measurement has evolved alongside the broader growth of modern investment management.

Early work in performance evaluation focused on distinguishing investment skill from investor cash-flow decisions. This objective naturally led to the development of time-weighted methodologies.

In parallel, discounted cash-flow techniques developed within corporate finance provided a framework for evaluating investment projects through the Internal Rate of Return. These methods were subsequently adapted to portfolio performance measurement and became known within the investment industry as Money-Weighted Returns or Dollar-Weighted Returns.

The practical challenges of computing exact IRRs motivated the development of approximation methodologies. Dietz introduced both the Simple Dietz and Modified Dietz Return estimators as practical methods for estimating money-weighted returns using limited valuation data [3].

The modern performance-evaluation literature developed alongside foundational contributions by Treynor [11] and Sharpe [10]. Although modern computing has largely removed the original computational constraints, Dietz methodologies remain widely used because of their simplicity, transparency, and interpretability.

1.3 Literature Review

The literature on portfolio performance measurement spans several related but often separate areas of research [1,4,5].

One branch focuses on investment performance evaluation and attribution. A second focuses on discounted cash-flow analysis and Internal Rate of Return methodologies. A third addresses continuous-time portfolio modeling and stochastic portfolio dynamics [6,9,12].

Despite extensive literature on these individual topics, relatively little attention has been devoted to developing a unified framework that places the major return methodologies within a common mathematical structure.

To the authors knowledge, the existing literature does not provide a unified continuous-time interpretation linking Time-Weighted Returns, Dietz estimators, Money-Weighted Returns, Internal Rates of Return, and AUM fee economics through the concept of average invested capital.

1.4 The Problem

The coexistence of multiple return methodologies raises several important questions:

1. Why do Time-Weighted Returns and Money-Weighted Returns sometimes produce substantially

different results?

2. Why does the Modified Dietz methodology frequently provide a close approximation to Money-Weighted Returns?

3. Under what conditions do the major return measures converge or diverge?

4. How should investors and investment managers interpret differences among reported performance measures?

1.5 Contributions of This Paper

The principal contribution of this paper is a unified continuous-time framework demonstrating how the major portfolio return measures used in practice may be interpreted as alternative reductions or approximations of a common portfolio process.

A second contribution is the development of a continuous-time interpretation of the Modified Dietz methodology. Specifically, Result 1 interprets the Modified Dietz denominator as a first-order approximation to average invested capital.

A third contribution is the extension of the framework to AUM fee arrangements.

Taken together, these contributions provide a coherent mathematical and economic framework for understanding portfolio performance measurement in both theory and practice.

1.6 Organization of the Paper

Section 2 introduces notation and mathematical preliminaries.

Section 3 develops the continuous-time portfolio framework.

Section 4 derives the principal return measures as reductions of the framework.

Section 5 develops the concept of average invested capital and presents Result 1.

Sections 6 and 7 examine approximation error and numerical illustrations.

Sections 8–10 discuss applications, implications, and conclusions.

2. Notation and Mathematical Preliminaries

Throughout the paper, portfolio performance is analyzed within a continuous-time framework.

Let

$$(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq 0},P)$$

denote a filtered probability space satisfying the usual conditions [6,9].

Let

$$W_t$$

denote a standard Brownian motion adapted to the filtration

$$\{\mathscr{F}_t\}_{t\geq 0}.$$

The portfolio value process is denoted by

$$V_t,$$

Where $V_t > 0$ represents the market value of the portfolio at time t .

External cash flows are represented by the cumulative process

$$C_t,$$

where positive increments correspond to contributions and negative increments correspond to withdrawals.

The expectation operator under the probability measure P is denoted by

$$E[\cdot].$$

Unless otherwise stated, all stochastic integrals are interpreted in the Itô sense [9].

2.1 Average Invested Capital

A central quantity in this paper is the average invested capital over the measurement horizon $[0,T]$.

This quantity is defined by

$$\bar{V}=\frac{1}{T}\int_0^T V_t\,dt. \tag{2.1}$$

The quantity $\bar{V}$ represents the time-average amount of capital exposed to investment risk during the measurement period.

As shown later, this quantity plays a fundamental role in:

- Money-Weighted Returns,
- Modified Dietz Return estimation,
- Internal Rate of Return approximations,
- AUM fee generation.

2.2 Portfolio Gain

Define portfolio gain over the interval $[0,T]$ by

$$G=V_T-V_0-\sum_{i=1}^{n}C_i, \tag{2.2}$$

where

$$C_1,\ldots,C_n$$

denote the external cash flows occurring during the measurement period.

Equation (2.2) removes the direct effect of contributions and withdrawals and therefore isolates investment gain or loss.

2.3 Return Measures

The paper considers four principal return measures:

1. Time-Weighted Return (TWR),
2. Internal Rate of Return (R_{IRR}),
3. Simple Dietz Return (SDR),
4. Modified Dietz Return (MDR),

The objective is not merely to compare these measures computationally, but rather to demonstrate that they arise as alternative reductions or approximations of a common continuous-time portfolio process.

2.4 Notation Summary

Symbol	Definition
V_t	Portfolio value at time t
V_0	Initial portfolio value
V_T	Terminal portfolio value
C_t	Cumulative cash-flow process
C_i	External cash flow at time t_i
W_t	Standard Brownian motion
μ_t	Instantaneous expected return
σ_t	Instantaneous volatility
G	Portfolio gain
$\bar{V}$	Average invested capital
R_{TWR}	Time-Weighted Return
R_{SD}	Simple Dietz Return
R_{MD}	Modified Dietz Return
R_{IRR}	Internal Rate of Return

3. Continuous-Time Portfolio Dynamics

Continuous-time portfolio models have a long history in mathematical finance and provide a natural framework for analyzing investment performance in the presence of external cash flows [6–9].

Let

$$\mu_t$$

denote the instantaneous expected return of the portfolio and let

$$\sigma_t$$

denote the instantaneous volatility.

The portfolio value process evolves according to [9]

$$dV_t = \mu_t V_t\, dt + \sigma_t V_t\, dW_t + dC_t. \tag{3.1}$$

Equation (3.1) states that portfolio value changes through three distinct mechanisms:

1. deterministic growth,
2. stochastic fluctuations,
3. external cash flows.

Integrating Equation (3.1) over the interval $[0,T]$ yields

$$V_T = V_0 + \int_0^T \mu_t V_t\, dt + \int_0^T \sigma_t V_t\, dW_t + \int_0^T dC_t. \tag{3.2}$$

Equation (3.1) expresses terminal portfolio value as the cumulative effect of investment growth, stochastic variation, and investor cash-flow decisions.

3.1 Economic Interpretation

The distinction between investment returns and external cash flows is fundamental to portfolio performance measurement.

The first two terms on the right-hand side of Equation (3.1) arise from investment activity.

The final term,

$$\int_0^T dC_t,$$

arises from investor decisions regarding contributions and withdrawals.

Different return methodologies differ primarily in how they treat this cash-flow component.

Time-Weighted Returns attempt to neutralize cash-flow effects in order to isolate manager performance.

Money-Weighted Returns and Internal Rates of Return incorporate cash-flow timing directly and therefore reflect the investor's realized experience.

Dietz estimators provide practical approximations to these money-weighted measures.

3.2 Continuous-Time Foundation for Return Measurement

Equation (3.1) provides the mathematical foundation for the remainder of the paper.

The central thesis is that commonly used return measures can be interpreted as alternative reductions or approximations of the same underlying portfolio process.

This perspective allows Time-Weighted Returns, Internal Rates of Return, and Dietz estimators to be analyzed within a common framework and clarifies the relationships among methodologies that are often presented separately in practice.

4. Return Measures as Reductions of the Framework

The continuous-time portfolio process developed in Section 3 provides a common foundation for the principal return measures used in investment practice.

The various methodologies differ primarily in how they treat external cash flows and invested capital.

4.1 Time-Weighted Returns

Time-Weighted Returns seek to measure investment performance independently of investor cash-flow decisions [2].

Let

$$P = \{t_0, t_1, \ldots, t_m\}$$

denote a valuation partition of the interval $[0,T]$.

Let

$$r_i = \frac{V_{t_i}^{+} - V_{t_{i-1}}^{-}}{V_{t_{i-1}}^{-}}$$

denote the return earned during subperiod i .

The Time-Weighted Return associated with partition P is

$$R_{TWR}(P) = \prod_{i=1}^{m}(1+r_i) - 1. \tag{4.1}$$

This is because the result depends upon the valuation partition,

$$R_{TWR} = R_{TWR}(P). \tag{4.2}$$

This observation becomes important in the analysis of valuation-partition effects developed later in the paper.

4.2 Simple Dietz Return

Using the portfolio gain defined in Equation (2.2), the Simple Dietz Return is

$$R_{SD} = \frac{G}{V_0 + \frac{1}{2}\sum_{i=1}^{n} C_i}. \tag{4.3}$$

The methodology assumes that all cash flows occur at the midpoint of the measurement period [3].

Although simple to compute, the approximation may become inaccurate when cash flows are large or occur unevenly through time.

4.3 Modified Dietz Return

Let

$$w_i = \frac{T - t_i}{T}$$

denote the fraction of the measurement period during which cash flow C_i remains invested.

The Modified Dietz denominator is

$$D_{MD} = V_0 + \sum_{i=1}^{n} w_i C_i. \tag{4.4}$$

The Modified Dietz Return is

$$R_{MD} = \frac{G}{D_{MD}}. \tag{4.5}$$

The timing weights allow the denominator to reflect invested capital through time more accurately than the Simple Dietz approximation.

4.4 Internal Rate of Return

The Internal Rate of Return is denoted by

$$R_{IRR}.$$

Within portfolio performance measurement, R_{IRR}. is commonly referred to as the Money-Weighted Return (MWR) or Dollar-Weighted Return (DWR). These terms represent the same underlying return concept.

The Internal Rate of Return satisfies

$$0 = -V_0 + \sum_{i=1}^{n} \frac{C_i}{(1+r)^{t_i}} + \frac{V_T}{(1+r)^T}. \tag{4.6}$$

Unlike Time-Weighted Returns, R_{IRR} incorporates both the timing and magnitude of investor cash flows and therefore reflects realized investor experience.

4.5 Comparison of Return Measures

The principal return measures may be viewed as addressing different economic questions.

Measure	**Primary Focus**
Time-Weighted Return	Manager performance
Internal Rate of Return	Investor experience
Simple Dietz Return	Approximate money-weighted performance
Modified Dietz Return	Improved approximation to money weighted performance

The differences among these measures arise not because they analyze different portfolios, but because they treat cash flows and invested capital differently.

5. Continuous-Time Interpretation of Invested Capital

The principal return measures examined in Section 4 differ primarily in their treatment of invested capital through time.

To develop a common framework, recall the definition of average invested capital introduced in Equation (2.1). The quantity $\overline{V}$ represents the average amount of capital exposed to investment risk during the measurement period and serves as the central quantity underlying the analysis that follows.

Result 1

Let

$$D_{MD} = V_0 + \sum_{i=1}^{n} w_i C_i \tag{5.1}$$

denote the Modified Dietz denominator.

Then

$$D_{MD} \approx \bar{V} \tag{5.2}$$

under a first-order approximation to invested capital through time.

Interpretation

Result 1 provides a continuous-time interpretation of the Modified Dietz denominator.

Rather than viewing the denominator as merely a computational device, Result 1 shows that it approximates average invested capital as defined in Equation (2.1).

This interpretation explains why the Modified Dietz Return frequently provides a close approximation to R_{IRR} in practice.

The result also establishes the conceptual bridge developed later between portfolio return measurement and AUM fee economics.

6. Approximation Error and Metric Divergence

Result 1 suggests that Modified Dietz returns approximate Internal Rates of Return because both methodologies depend upon invested capital through time.

However, agreement among return measures should not be expected under all circumstances.

This section identifies two distinct sources of divergence.

6.1 Invested-Capital Approximation Error

The first source of divergence arises from approximation error in invested capital.

The Modified Dietz denominator

$$D_{MD} = V_0 + \sum_{i=1}^{n} w_i C_i \tag{6.1}$$

provides a first-order approximation to average invested capital.

As established in Result 1,

$$D_{MD} \approx \bar{V}. \tag{6.2}$$

When portfolio values evolve smoothly and cash flows are moderate, the approximation is typically accurate.

However, approximation error may increase when:

- cash flows are large,
- cash flows are concentrated in time,
- portfolio volatility is high,
- valuation intervals are long.

Under such conditions, Modified Dietz returns may diverge from R_{IRR}.

6.2 Valuation-Partition Error

A second and conceptually distinct source of divergence arises from the valuation partition used in Time-Weighted Return calculations.

Recall from Equation (4.2) that

$$R_{TWR} = R_{TWR}(P), \tag{6.3}$$

where

$$P = \{t_0, t_1, \ldots, t_m\}$$

denotes the valuation partition.

Time-Weighted Returns are therefore not solely functions of portfolio performance. They are also functions of the valuation schedule.

When valuations occur immediately before and after every external cash flow, partition error is minimized.

When valuations are infrequent, reported Time-Weighted Returns may differ from those obtained using a finer partition.

6.3 Two Sources of Divergence

The distinction between invested-capital approximation error and valuation-partition error is important.

The first affects comparisons between:

- Modified Dietz Returns,
- Internal Rates of Return.

The second affects comparisons between:

- Time-Weighted Returns,
- Money-Weighted Returns.

These are fundamentally different mechanisms.

Accordingly, disagreement among return measures should not automatically be interpreted as computational error.

Instead, divergence may reflect differences in:

1. cash-flow timing,
2. invested-capital approximation,
3. valuation frequency,
4. portfolio volatility.

6.4 Limiting Behavior

As valuation frequency increases and cash-flow intervals become small, partition effects diminish.

Likewise, as invested-capital approximations become increasingly accurate,

$$D_{MD} \rightarrow \overline{V}. \tag{6.4}$$

Under these conditions, the principal return measures tend to converge.

Conversely, divergence becomes most pronounced when:

- cash flows are large,
- cash flows are irregular,
- valuation frequency is low,
- portfolio volatility is high.

6.5 Implications

The analysis suggests that no single return measure is universally superior.

Rather, each methodology answers a different economic question.

Time-Weighted Returns emphasize manager performance.

Internal Rates of Return emphasize investor experience.

Dietz methodologies provide practical approximations based upon invested capital through time.

Understanding the source of divergence among these measures is therefore more informative than simply observing that the reported returns differ.

7. Numerical Illustrations

The purpose of this section is to illustrate the behavior of the principal return measures under alternative cash-flow patterns.

The examples demonstrate both convergence and divergence among:

- Time-Weighted Returns,
- Internal Rates of Return,
- Modified Dietz Returns.

7.1 Example 1: Moderate Cash Flows

Consider a portfolio with

$$V_0 = 100.$$

At time

$$t = \frac{1}{2},$$

an external contribution of

$$C_1 = 50$$

is made.

At year-end,

$$V_T = 190.$$

Portfolio gain is

$$G = 190\text{-}100\text{-}50 = 40. \tag{7.1}$$

The timing weight is

$$w_1 = 0.5. \tag{7.2}$$

The Modified Dietz denominator becomes

$$D_{MD} = 100{+}0.5(50) = 125. \tag{7.3}$$

Therefore

$$R_{MD} = \frac{40}{125} = 32.00\%. \tag{7.4}$$

The Internal Rate of Return satisfies

$$0 = -100 - \frac{50}{(1+r)^{0.5}} + \frac{190}{1+r}, \tag{7.5}$$

yielding

$$R_{IRR} \approx 31.7\%. \tag{7.6}$$

The close agreement reflects minimal invested-capital approximation error.

7.2 Example 2: Large Early Contribution

Let

$$V_0 = 100,$$

and suppose a contribution of

$$C_1 = 200$$

occurs at

$$t = 0.20.$$

At year-end,

$$V_T = 360.$$

Portfolio gain is

$$G = 360\text{-}100\text{-}200 = 60. \tag{7.7}$$

The timing weight is

$$w_1 = 0.80. \tag{7.8}$$

The Modified Dietz denominator becomes

$$D_{MD} = 100{+}0.80(200) = 260. \tag{7.9}$$

Therefore

$$R_{MD} = \frac{60}{260} = 23.08\%. \tag{7.10}$$

The Internal Rate of Return satisfies

$$0 = -100 - \frac{200}{(1+r)^{0.20}} + \frac{360}{1+r}, \tag{7.11}$$

yielding

$$R_{IRR} \approx 22.6\%. \tag{7.12}$$

The difference remains small despite the large contribution.

7.3 Example 3: Time-Weighted Return Divergence

Assume

$$V_0 = 100.$$

At time

$$t = 0.25,$$

an investor contributes

$$200.$$

Immediately before the contribution the portfolio value has risen to

$$120.$$

Immediately after the contribution the value becomes

$$320.$$

At year-end,

$$V_T = 400.$$

The first subperiod return is

$$r_1 = \frac{120-100}{100} = 20\%. \quad (7.13)$$

The second subperiod return is

$$r_2 = \frac{400-320}{320} = 25\%. \quad (7.14)$$

Thus

$$R_{TWR} = (1.20)(1.25)\text{-}1\text{=}50.0\%. \quad (7.15)$$

Portfolio gain is

$$G = 400\text{-}100\text{-}200 = 100. \quad (7.16)$$

The timing weight is

$$w_1 = 0.75. \quad (7.17)$$

The Modified Dietz denominator becomes

$$D_{MD} = 100\text{+}0.75(200) = 250. \quad (7.18)$$

Therefore

$$R_{MD} = \frac{100}{250} = 40.0\%. \quad (7.19)$$

The divergence reflects differences in economic interpretation rather than computational error see **Table 7.1.**

Table 7.1 Convergence and Divergence of Portfolio Return Measures Under Alternative Cash-Flow Scenarios

Example	Cash-Flow Scenario	Conceptual Lesson	Modified Dietz Return	R_{IRR}	TWR	Interpretation
Convergence						
1	Mid-period contribution	Modified Dietz Return closely approximates R_{IRR}	32.0%	31.7%	—	Minimal invested-capital approximation error
2	Large early contribution	Timing effects increase, but The Modified Dietz Return remains close to R_{IRR}	23.1%	22.6%	—	Approximation remains robust despite larger cash flow
Divergent Region						
3	Contribution after appreciation	Time-Weighted and Money-Weighted methodologies separate materially	40.0%	≈39–40%	50.0%	Valuation-partition and economic-interpretation effects dominate

Interpretation: Examples 1 and 2 illustrate convergence between the Modified Dietz Return and the Internal Rate of Return when invested-capital approximation error remains small. Example 3 illustrates divergence between Time-Weighted and Money-Weighted methodologies resulting from cash-flow timing and valuation-partition effects. The divergence reflects differences in economic interpretation rather than computational error.

8. Extension to AUM Fees

The preceding sections established that average invested capital plays a central role in portfolio return measurement. In particular, Result 1 demonstrated that the Modified Dietz denominator may be interpreted as a first-order approximation to average invested capital through time.

The same quantity also arises naturally in many AUM fee arrangements.

Consequently, average invested capital provides a common mathematical link between portfolio performance measurement and investment-management economics.

8.1 Assets Under Management Fee Structures

Many investment advisers, wealth managers, pension consultants, and institutional asset managers charge fees based upon assets under management.

Let

$$f$$

denote the annual fee rate expressed as a decimal fraction.

Examples include

$$f = 0.01$$

for a 1% annual fee and

$$f = 0.005$$

for a 0.50% annual fee.

Because fee revenue depends upon the capital entrusted to management, the fee-generating process is naturally linked to invested capital through time.

8.2 Continuous-Time Fee Generation

Suppose the portfolio evolves according to [9]

$$dV_t = (\mu_t - f)V_t dt + \sigma_t V_t dW_t + dC_t \quad (8.1)$$

Equation (8.1) modifies the portfolio dynamics of Section 3 by introducing a management fee rate f.

The fee reduces portfolio growth while generating revenue to the investment manager.

8.3 Total Fee Revenue

Let F_T denote cumulative fee revenue earned during the interval $[0,T]$.

The total fee revenue is

$$F_T = \int_0^T f V_t \, dt. \quad (8.2)$$

Assuming a constant fee rate,

$$F_T = f \int_0^T V_t \, dt. \quad (8.3)$$

Using Equation (2.1), we obtain

$$F_T = f T \overline{V} \quad (8.4)$$

Equation (8.4) is the fundamental fee-revenue identity.

8.4 Interpretation

Equation (8.4) demonstrates that fee revenue depends directly upon average invested capital.

This result mirrors the interpretation developed earlier for the Modified Dietz denominator.

Specifically,

$$D_{MD} \approx \overline{V}$$

from Result 1, while

$$F_T = f T \overline{V}.$$

Thus both portfolio return measurement and fee generation depend upon the same underlying quantity:

$$\overline{V}$$

8.5 Economic Significance

The analysis suggests that average invested capital serves as a unifying concept across several areas of investment practice.

Within performance measurement, average invested capital underlies:

- Modified Dietz Return estimation,
- Money-Weighted Return approximations,
- return-measure comparisons.

Within investment-management economics, average invested capital determines:

- fee revenue,
- revenue stability,
- the economic value of client relationships.

Consequently, the same quantity that governs the approximation properties of return measures also governs the economics of fee-based investment management.

9. Discussion and Practical Implications

The preceding sections developed a unified continuous-time framework for portfolio return measurement and demonstrated how Time-Weighted Returns, Money-Weighted Returns, Internal Rates of Return, and Dietz estimators may be interpreted as alternative reductions or approximations of a common portfolio process.

The analysis identified average invested capital as a central quantity linking these methodologies and subsequently extended the framework to AUM fee arrangements.

This section discusses the practical implications of these results.

9.1 Manager Evaluation

Time-Weighted Returns are widely regarded as the preferred methodology for evaluating investment managers because they seek to eliminate the effects of investor cash-flow decisions [2].

When external cash flows are beyond the control of the manager, the Time-Weighted Return provides a measure of performance attributable primarily to investment decisions.

Accordingly, Time-Weighted Returns remain particularly appropriate when the objective is to assess manager skill rather than investor experience.

9.2 Investor Evaluation

From the investor's perspective, however, cash-flow timing is often an important component of realized investment outcomes.

Money-Weighted Returns and Internal Rates of

Return incorporate both the timing and magnitude of external cash flows and therefore reflect the investor's actual experience.

Accordingly, differences between Time-Weighted and Money-Weighted Returns should not be viewed as inconsistencies. Rather, they arise because the measures are designed to evaluate different aspects of portfolio performance.

9.3 Interpretation of Metric Differences

A common source of confusion in performance reporting occurs when different return measures produce materially different results.

The analysis of Section 6 suggests that such differences frequently arise from identifiable economic and mathematical factors rather than computational error.

In particular, divergence may result from:

1. Cash-flow timing,
2. Cash-flow magnitude,
3. Invested-capital approximation error,
4. Valuation-partition selection,
5. Portfolio volatility.

Understanding the source of divergence is often more informative than merely observing that reported returns differ.

Consequently, practitioners should interpret return measures in the context of the economic question being addressed rather than treating any single methodology as universally superior.

9.4 Fiduciary and Reporting Considerations

Portfolio return measurement plays an important role in fiduciary oversight, pension administration, endowment management, and wealth-management reporting.

Different stakeholders may require different performance measures depending upon their objectives.

The framework developed here suggests that no single return measure can simultaneously satisfy all reporting objectives.

Instead, performance measurement should be aligned with the specific decision context under consideration.

Because different return methodologies may produce materially different numerical results for the same portfolio, effective performance reporting requires clear disclosure of the methodology employed. While the investor's actual wealth accumulation is independent of the reporting metric selected, the interpretation of performance may depend strongly upon whether returns are presented on a time-weighted, money-weighted, or Dietz basis. Consequently, fiduciaries, investment managers, and clients should understand both the reported return and the economic question that the metric is intended to answer.

9.5 Average Invested Capital as a Unifying Concept

One of the principal findings of this paper is the emergence of average invested capital, $\overline{V}$, as defined in Equation (2.1), as a unifying quantity.

This quantity appears naturally in:

- Money-Weighted Return approximations,
- Modified Dietz Return estimation,
- Internal Rate of Return analysis,
- AUM fee generation.

The repeated appearance of average invested capital suggests that it occupies a more fundamental role in portfolio analysis than is often recognized in practice.

Rather than viewing return measures as unrelated computational procedures, the present framework interprets them as alternative methods for analyzing a common underlying quantity.

9.6 Return Measurement and Fee Interpretation

Because management fees are generally assessed on assets under management, fee revenue depends upon invested capital through time.

By contrast, portfolio performance may be reported using Time-Weighted Returns, Money-Weighted Returns, or Dietz-based methodologies, each of which emphasizes a different economic perspective.

Consequently, identical portfolios may exhibit different reported performance depending upon the return methodology employed, even though fee revenue remains linked to the same invested-capital base.

This observation does not imply that any particular return measure is preferable. Rather, it highlights the importance of understanding the economic question addressed by the reported metric when evaluating both investment performance and management costs.

The relationship between investment performance, assets under management, and management compensation has long been debated within both the investment industry and the academic literature. Different compensation structures embody different incentive arrangements, and no single fee model has achieved universal acceptance. Equation (8.1) further illustrates that management fees enter the portfolio dynamics as a reduction in the portfolio growth rate, thereby affecting investor net returns while simultaneously generating manager revenue. The framework developed here does not evaluate the relative merits of alternative compensation models; rather, it clarifies the role of average invested capital in fee generation under assets under management arrangements.

9.7 Broader Perspective

The framework developed in this paper unifies several return methodologies that are often presented separately within the investment literature.

By placing these measures within a common continuous-time setting, the analysis provides a clearer understanding of their relationships, assumptions, and areas of applicability.

The framework also illustrates how concepts originating in stochastic portfolio theory may be connected directly to practical performance measurement and investment-management economics.

10. Conclusions

This paper developed a unified continuous-time framework for portfolio return measurement in the presence of external cash flows.

Beginning with a continuous-time portfolio process, the analysis demonstrated how Time-Weighted Returns, Money-Weighted Returns, Internal Rates of Return, and Dietz Return estimators may be interpreted as alternative reductions or approximations of a common underlying portfolio evolution.

A principal contribution of the paper is the identification of average invested capital, $\overline{V}$, as defined in Equation (2.1), as a unifying quantity underlying several widely used return methodologies. Result 1 established that the Modified Dietz denominator may be interpreted as a first-order approximation to this quantity.

This interpretation provides a theoretical explanation for the practical effectiveness of the Modified Dietz methodology and clarifies its relationship to R_{IRR} .

The analysis further demonstrated that differences among return measures arise from identifiable economic and mathematical mechanisms. In particular, the paper distinguished between invested-capital approximation error and valuation-partition effects, thereby providing a framework for understanding both convergence and divergence among commonly reported performance measures.

Numerical examples illustrated these effects and showed how differences in cash-flow timing, cash-flow magnitude, valuation frequency, and portfolio evolution may influence reported returns.

The framework was subsequently extended to AUM fee arrangements. This extension established that fee revenue depends upon the same average invested-capital quantity that underlies several return-measurement methodologies, thereby linking portfolio performance measurement and investment-management economics within a common mathematical structure.

More broadly, the paper suggests that return measures should not be viewed merely as alternative computational procedures. Rather, they represent different perspectives on portfolio performance and are designed to answer different economic questions. Time-Weighted Returns emphasize manager performance, whereas Money-Weighted Returns and Dietz methodologies emphasize the investor's experience through time.

The unified framework developed here provides a clearer understanding of these relationships and offers a common foundation for future work in portfolio performance measurement, investment reporting, fiduciary oversight, and fee-based investment management.

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